

MATH 1650: GRAPHS OF RATIONAL FUNCTIONS SUMMARY

EXAMPLE: Let $f(x) = \frac{x^2 + 6x + 9}{x^2 - 9}$.

- Find the values excluded from the domain: Solving $x^2 - 9 = 0$ gives $x = \pm 3$.
- State the domain using interval notation: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.
- Reduce $f(x)$ to lowest terms: $f(x) = \frac{x^2 + 6x + 9}{x^2 - 9} = \frac{(x+3)(x+3)}{(x-3)(x+3)} = \frac{x+3}{x-3}$, provided $x \neq -3$.
- List the vertical asymptotes, if any: VA: $x = 3$
- Find the holes in the graph, if any: Hole when $x = -3$: $f(-3) \approx \frac{-3+3}{-3-3} = \frac{0}{-6} = 0$. Hole at $(-3, 0)$.
- Find the y-intercept, if any: $x = 0$, $y = f(0) = \frac{0+3}{0-3} = -1$. y-int: $(0, -1)$.
- Find the x-intercepts, if any: solve $y = f(x) = \frac{x+3}{x-3} = 0$ gives $x+3 = 0$ or $x = -3$.
However, since $x = -3$ is not in the domain of f , there are no x-intercepts.
- Find the horizontal or slant asymptote, if any: as $x \rightarrow \pm\infty$, $f(x) = \frac{x+3}{x-3} \approx \frac{x}{x} = 1$, so $y = 1$ is the HA.
- Make a Sign Diagram for $f(x)$.

$$\begin{array}{ccccccc} (+) & ? & (-) & ? & (+) & f(x) & \\ \leftarrow & & & & & & \\ & -3 & & 3 & & & x \end{array}$$

- Graph $y = f(x)$.

